LIMIT FUNGSI TRIGONOMETRI

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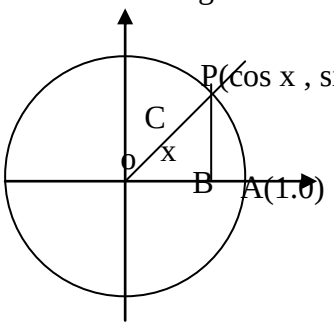
Perhatikan gambar berikut :



Untuk nilai Limit fungsi trigonometri ,yang menghasilkan bentuk $\frac{0}{0}$ (tak tentu)

untuk $x \to 0$,digunakan rumus-rumus sehingga didapat nilai limitnya.Dari gambar diatas 0 adalah titik pusat lingkaran yang berjari-jari r = 1 satuan , $\angle$ AOP = x radian.
Luas juring OBC $\leq$ *luas* $\Delta OBP \leq$ luas juring OAP

$$\frac{1}{2}(OB)^2.x \leq \frac{1}{2}OB.PB \leq \frac{1}{2}(OA)^2.x$$

$$\frac{1}{2}.x.Cos^2 x \leq \frac{1}{2}\cos x.\sin x \leq \frac{1}{2}(1)^2.x \qquad \text{masing-masing ruas } \times(\frac{2}{x.\cos x})$$

$$\cos x \quad \leq \quad \sin x \quad \leq \frac{1}{\cos x} \qquad \text{jika } x \to 0 \text{ ,maka}$$

$$1 \quad \leq \lim_{x\to 0}\frac{\sin x}{x} \quad \leq \quad 1$$

- Sehingga diperoleh kesimpulan bahwa : $\lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{x}{\sin x} = 1$
- Sedangkan untuk $\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\sin x}{x.\cos x} = 1.1 = 1$

$$\lim_{x\to 0}\frac{x}{\tan x} = \lim_{x\to 0}\frac{x.\cos x}{\sin x} = 1.1 = 1$$

Sehingga diperoleh kesimpulan bahwa : $\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{x}{\tan x} = 1$

- Dengan menggunakan pemisalan $x - a = u$, maka diperoleh:
  $\lim_{x\to a}\frac{\sin(x-a)}{x-a} = \lim_{u\to 0}\frac{\sin u}{u} = 1$ dan $\lim_{x\to a}\frac{\tan(x-a)}{x-a} = \lim_{u\to 0}\frac{\tan u}{u} = 1$

Contoh 1:
Selesaikan

1. $\lim_{x\to 0}\cos 5x$	2. $\lim_{x\to \frac{\pi}{4}}\frac{\tan x}{x}$
3. $\lim_{x\to 0}\frac{\sin 2x}{\tan 3x}$	4. $\lim_{x\to \frac{\pi}{3}}\frac{\sin(x-\frac{\pi}{3})}{\tan(3x-\pi)}$

5. $\lim_{x\to 0} \frac{\cot 4x}{\cot x}$	6. $\lim_{x\to 0} \frac{\operatorname{cosec} 3x}{\operatorname{cosec} 5x}$
7. $\lim_{x\to \pi} \frac{\sin x}{\tan x}$ (gunakan rumus sudut berelasi)	8. $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cot x}$ (gunakan rumus sudut berelasi)

Dari contoh di atas dapat konsep penyelesian limit fungsi trigonometri sebagai berikut :
Nilai limit $\lim_{x \to a} f(x)$, dengan f(x) terdapat fungsi trigonometri adalah : dengan mengganti x dengan a atau f(a)

- jika f(a) terdefinisi maka $\lim_{x \to a} f(x)$ = f(a)
- Jika $f(a) = \frac{0}{0}$ (tak tentu), jika merupakan salah satu bentuk dari

  $\lim_{x\to a} \frac{\sin m(x-a)}{n(x-a)}$, $\lim_{x\to a} \frac{m(x-a)}{\sin n(x-a)}$, $\lim_{x\to a} \frac{\tan m(x-a)}{n(x-a)}$, $\lim_{x\to a} \frac{m(x-a)}{\tan n(x-a)}$

  $\lim_{x\to a} \frac{\sin m(x-a)}{\sin n(x-a)}$, $\lim_{x\to a} \frac{sim(x-a)}{\tan n(x-a)}$, $\lim_{x\to a} \frac{\sin m(x-a)}{\tan n(x-a)}$, atau $\lim_{x\to a} \frac{\tan m(x-a)}{\sin n(x-a)}$,

  maka $\lim_{x \to a} f(x) = \frac{m}{n}$. Apabila $f(a) = \frac{0}{0}$ bukan merupakan bentuk di atas, maka ubahlah soal sehingga menjadi bentuk rumus di atas dengan menggunakan : pemisalan, pemfaktoran, rumus sudut berelasi, rumus trigonometri sudut rangkap, atau penjumlah fungsi trigono menjadi perkalian.
  Rumus-rumus trigonometri antara lain :

• $\sin(\pi - x) = \sin x$ • $si(\pi - x) = \sin x$ • $\cos(\frac{\pi}{2} - x) = \cos(\frac{3\pi}{2} + x) = \sin x$ • $\cot(\frac{\pi}{2} - x) = \cot(\frac{3\pi}{2} - x) = \tan x$ • $\cos(\frac{\pi}{2} + x) = \cos(\frac{3\pi}{2} - x) = -\sin x$ • $\cot(\frac{\pi}{2} + x) = \cot(\frac{3\pi}{2} + x) = -\tan x$ • $\sin^2 x + \cos^2 x = 1$	• Sin 2x = 2sinx cosx • Cos 2x = $\cos^2$ x - $\sin^2$ x = 1- 2 $\sin^2$ x atau $1 - \cos 2x = 2\sin^2 x$ • $\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$ • $\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$ • $\tan A - \tan B = \tan(A-B)(1+\tan A.\tan B)$

Contoh 2
Selesaiakan

1. $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cot x}$	2. $\lim_{x\to 0} \frac{1-\cos 4x}{x \tan 3x}$

3. $\lim_{x\to 0} \dfrac{\sin 2x - 2\sin x}{x^3}$	4. $\lim_{x\to \frac{\pi}{3}} \dfrac{\sin 3x}{\left(x - \frac{\pi}{3}\right)}$
5. $\lim_{x\to \frac{\pi}{2}} \dfrac{1-\sin x}{\cot x}$	6. $\lim_{x\to 1} \dfrac{\sin \pi x}{x^2 - 1}$
7. $\lim_{x\to 0} \dfrac{\sin x + \tan 3x - 2x}{\tan 4x - \sin 5x + 6x}$	8. $\lim_{x\to 0} \dfrac{\cos 3x - \cos x}{1 - \cos^2 2x}$

Latihan 4.
Selesaiakan

1. $\lim_{x\to \frac{\pi}{2}} \dfrac{2x}{\sin x} = \ldots$	2. $\lim_{x\to 3\pi} \cos^2 x + \sin^2 x = \ldots$

3. $\lim\limits_{x\to 0}\dfrac{3\sin 4x}{5\tan 6x}=\ldots$	4. $\lim\limits_{x\to 0}\dfrac{2x.\sin x}{1-\cos 2x}=\ldots$
5. $\lim\limits_{x\to a}\dfrac{(2a+x)\sin(a-x)}{x^2-a^2}=\ldots$	6. $\lim\limits_{x\to a}\dfrac{\sin x-\sin a}{2x-2a}=\ldots$
7. $\lim\limits_{x\to 0}\dfrac{\sin x-\sin x.\cos x}{4x^3}$	8. $\lim\limits_{x\to -a}\dfrac{\cos x-\cos a}{x+a}=\ldots$
9. $\lim\limits_{x\to b}\dfrac{2b-2x}{3x-3b+\tan(x-b)}=\ldots$	10. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\cos x-\sin x}{\cos 2x}=\ldots$
11. $\lim\limits_{x\to 0}\dfrac{6x^3}{\cos x.\sin 3x-\sin 3x}=\ldots$	12. $\lim\limits_{x\to 0}\dfrac{2-2\cos 4x}{\cos 2x-1}=\ldots$

13. $\lim\limits_{x\to 0}\dfrac{2x^2}{\sin 2x}=\ldots$	14. $\lim\limits_{x\to 3}\dfrac{(x+1).\sin(3-x)}{x^2-3}=\ldots$
15. $\lim\limits_{x\to 0}\dfrac{2\cos 4x-2}{3x^2}=\ldots$	16. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin x}=\ldots$
17. $\lim\limits_{x\to 0}\dfrac{a\sin x+b\tan x}{x\sqrt{ab}}=\ldots$	18. $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{x^2.\tan x}=\ldots$
19. $\lim\limits_{x\to 0}\dfrac{\tan^3 2x}{x.\sin^2 2x}=\ldots$	20. $\lim\limits_{x\to 0}\dfrac{\cos x-1}{4x^2}=\ldots$
21. $\lim\limits_{x\to \pi}\dfrac{\left(\sin\frac{1}{4}x-\cos\frac{1}{4}x\right)^2}{\cos^2\frac{1}{2}x}=\ldots$	22. $\lim\limits_{x\to 0}\dfrac{\tan^2 x+\sin x}{\sin^2 x+\tan x}=\ldots$

23. $\lim\limits_{x\to 3}\dfrac{x^4-9x^2}{\sin\left(9-x^2\right)}=\ldots$	24. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan x}{\sin x-\cos x}=\ldots$
25. $\lim\limits_{x\to 0}\dfrac{\sin x-\tan x}{x^3}=\ldots$	26. $\lim\limits_{x\to 0}\dfrac{1-\cos^3 x}{1-\cos^2 x}=\ldots$
27. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1+\cos 2x}{\cos x}=\ldots$	28. $\lim\limits_{x\to\sqrt{a}}\dfrac{\left(a+x^2\right)\tan\left(x-\sqrt{a}\right)}{x^2-a}=\ldots$
29. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{1+\sin x}}{1-\sqrt{1-\sin x}}=\ldots$	30. $\lim\limits_{x\to\pi}\dfrac{1+\cos x}{(x-\pi)\sin x}=\ldots$

31. $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{2\sin x.\cos x-1}{6\cos^2 x-3}=\ldots$	32. $\lim\limits_{x\to 1} \dfrac{1-x^2}{\tan(\pi x)}=\ldots$
33. $\lim\limits_{x\to 0} \dfrac{\sin 5x-six4x-\sin 2x+\sin x}{\sin 3x-3\sin x}=\ldots$	34. $\lim\limits_{x\to 0} \dfrac{1-\cos 3x.\cos x}{x^2}=\ldots$
35. $\lim\limits_{x\to 1} \dfrac{x^2-1+\sin(x^2+x-2)}{x^2-3x+2+\tan(1-x)}=\ldots$	36. $\lim\limits_{x\to 0} \dfrac{\sin 5x-six4x-\sin 2x+\sin x}{\sin 3x-3\sin x}=\ldots$